

Nuclear Bound States of Antikaons, or Quantized Multiskyrmions?

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Abstract

The spectrum of strange multibaryons is considered within the chiral soliton model using one of several possible $SU(3)$ quantization models (the bound state rigid oscillator version). The states with energy below that of antikaon and corresponding nucleus can be interpreted as antikaon-nucleus bound states. In the formal limit of small kaon mass the number of such states becomes large, for real value of this mass there are at least several states with positive and negative parity in the energy gap of one kaon mass. For large values of binding energies interpretation of such states just as antikaon-nuclear bound states becomes more ambiguous.

1 Introduction

The studies of multibaryon states with different values of flavor quantum numbers are of permanent interest. They are closely related to the problem of existence of strange quark matter and its fragments, strange stars (analogs of neutron stars). Besides traditional approaches to this problem based usually on the potential and/or quark models, the chiral $SU(3)$ dynamics, mean field theories, etc., the chiral soliton approach (CSA) proposed by Skyrme [1] is effective and has certain advantages before conventional methods (some early descriptions of this model can be found in [2]). The quantization of the model performed first in the $SU(2)$ configuration space for the baryon number one states [3], somewhat later for configurations with axial symmetry [4, 5] and for multiskyrmions [6, 7, 8], allowed, in particular, to describe the properties of nucleons and Δ -isobar [3] and, more recently, some properties of light nuclei, including so called "symmetry energy" [9]³ and many other properties [11].

The $SU(3)$ quantization of the model has been performed within the rigid [12] or soft [13] rotator approach and also within the bound state model [14]. The binding energies of the ground states of light hypernuclei have been described within a version of the bound state chiral soliton model [15], in qualitative, even semiquantitative agreement with empirical data [16]. The collective motion contributions have been taken into account here (single particles excitations

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³Recently the neutron rich isotope ^{18}B has been found to be unstable relative to the decay $^{18}B \rightarrow ^{17}B + n$ [10], in agreement with prediction of the CSA [9]. This can be considered as an illustration of the fact that the CSA provides quite realistic predictions for the case of nonstrange nuclei.

should be added), and special subtraction scheme has been used to remove uncertainties in absolute values of masses intrinsic to the CSA. It makes sense therefore to extend such investigation to the higher in energy (excited) states, some of them may be interpreted as antikaon-nuclei bound states.

The antikaon-nuclei interactions and possible bound states of antikaons and nuclei have attracted recently much attention [17] - [26]. Theoretically deeply bound states of antikaons in nuclei have been obtained as a solution of many-body problem by Akaishi and Yamazaki [17]-[18]. Most recent reviews of this topic within the framework of conventional approaches can be found in [28, 29]. Here we investigate the possibility of interpretation of such states as quantized multiskyrmions (configuration with baryon number one is called usually a skyrmion). The spectrum of quantized multiskyrmions is very rich, and some of these states are appropriate for interpretation as bound antikaon-nuclei states.

Within the CSA there is a simple argument that at small value of the kaon mass m_K there should be quantized states of multiskyrmions with the mass below the sum of masses of the kaon and corresponding number of nucleons. Indeed, the strangeness (flavor) excitation energies are proportional to m_K^2 , both in the rotator [12, 13] and in the bound state models of skyrmion quantization [14]. Therefore, the mass of any state with baryon number B , strangeness S , isospin I , spin J can be presented as sum of two terms

$$M(B, S, I, J...) \simeq M(B, S = 0, ..) + m_K^2 \Gamma_B C(B, S, I, J...), \quad (1)$$

where Γ_B is the Σ -term (see Table 1), $C(B, S, I, J...)$ is some quantity of the order ~ 1 , depending on quantum numbers of the system. Evidently, at small enough m_K the contribution given by (1) is smaller than the sum $M(B, S = 0, ..) + |S|m_K$, and the number of states with the mass given by (1) in the gap between $M(B, S = 0, ..)$ and $M(B, S = 0, ..) + |S|m_K$ becomes large. This argument is quite rigorous, however, for realistic value of m_K it is a question of numerical calculation to find out, which states have the energy below that of the multibaryon plus antikaon system (here we consider the case of strangeness $S = -1$).

The interpretation of these states with fixed external quantum numbers in terms of hadronic constituents is not straightforward and not unique. Each state is the whole Fock column of hadronic components with different weights. We could only, in some particular situations, make statements about dominance of some components of this Fock column.

It should be specially pointed out that here we are using one of possible $SU(3)$ quantization models, the rigid oscillator version of the bound state model [15] which seems to be the simplest one. This quantization scheme can provide quantized states with definite restrictions on allowed quantum numbers of the states, including their spatial parity. E.g., only positive parity baryons appear when the basic baryon number 1 hedgehog-type $SU(2)$ configuration is quantized in this way. To get the states with negative parity, for example, the low mass $\Lambda(1405)$ state, an actual candidate to be the antikaon-nucleon bound state, one should provide at least second order expansion in mesonic fluctuations around the basic classical configuration (hedgehog). Considerable success in describing the properties of $\Lambda(1405)$ has been reached in this way in [30].

For the case of multiskyrmions similar approach is technically very complicated and is not performed except few attempts [31, 32]. In the $SU(2)$ case the qualitative description of some dibaryon states was obtained in [32]. Therefore, the expected spectrum of negative strangeness states may be considerably richer than obtained in present paper.

In the next section isotopical properties of the $\bar{K}NN$ and $\bar{K}NNN$ systems are briefly discussed. Section 3 contains description of starting positions of the CSA, in section 4 we recollect the spectrum of $SU(2)$ quantized dibaryons, section 5 contains the formulas summarizing the

CSA results for strange (flavored) multiskyrmions, our main results for the spectrum of strange baryonic states with $B = 2$ and 3 are presented in sections 6,7. Our former results for strange dibaryons are recollected in section 6. Excitations of the ground states of the $B = 2$ and 3 systems in some cases could be interpreted as antikaon-nuclei bound states.

2 Phenomenology

The K^-pp cluster has been proposed in [18] as a fundamental unit which plays an important role in formation of similar strangeness $S = -1$ clusters in heavier nuclei.

Here we discuss first some consequences of isotopic invariance of strong interactions involving strange particles. The state K^-pp which has the 3-d component of isospin $I_3 = 1/2$, is in fact a coherent combination of states with isospins $I = 3/2$ and $I = 1/2$:

$$|K^-pp\rangle = \sqrt{\frac{1}{3}}|\bar{K}NN; 3/2, +1/2\rangle + \sqrt{\frac{2}{3}}|\bar{K}NN; 1/2, +1/2\rangle. \quad (2)$$

Another physical state with same quantum numbers is

$$|\bar{K}^0(pn)_{I=1}\rangle = \sqrt{\frac{2}{3}}|\bar{K}NN; 3/2, +1/2\rangle - \sqrt{\frac{1}{3}}|\bar{K}NN; 1/2, +1/2\rangle, \quad (3)$$

where $(pn)_{I=1}$ system has isospin $I = 1$. So, same cluster which can be seen in K^-pp system should be seen also in $\bar{K}^0(pn)$ system, but with about 4 times smaller probability⁴. The $\bar{K}NN$ state with isospin $I = 3/2$ includes the state with charge +2, it is \bar{K}^0pp , and state with charge -1, it is K^-nn .

Another possibility to have the state with isospin $I = 1/2$ is to combine antikaon state with the isospin zero $2N$ state:

$$|\bar{K}NN; 1/2, +1/2\rangle = |\bar{K}\rangle|(pn)_{I=0}\rangle. \quad (4)$$

In total, we have for the $\bar{K}NN$ system 8 different components which can be splitted into quartet (isospin $I = 3/2$) and two doublets. Within the CSA we shall obtain the states with baryon number 2 and quantum numbers — strangeness, isospin, spin — as indicated above, and estimate their masses.

Similar for the $B = 3$ systems. In the case of $\bar{K}NNN$ system we have in total 16 components which can be separated into one quintet with maximal isospin $I = 2$, three triplets with $I = 1$ and two singlets. The maximal value of the 3-d component of isospin is $I_3 = +2$ (\bar{K}^0ppp -system), and minimal value is $I_3 = -2$ (K^-nnn system). As it was shown previously and we shall see here, within the CSA there is specific dependence of the mass of baryonic system on its isospin, usually states with lower isospin have smaller energy.

3 Basic ingredients and features of the CSA

The CSA is based on few principles and ingredients incorporated in the *truncated* effective chiral lagrangian [1, 2, 3]:

$$L^{eff} = -\frac{F_\pi^2}{16}Tr l_\mu l_\mu + \frac{1}{32e^2}Tr[l_\mu l_\nu]^2 + \frac{F_\pi^2 m_\pi^2}{8}Tr(U + U^\dagger - 2) + ..., \quad (5)$$

⁴We take into account that the pn system has isospin $I = 1$ with probability 1/2.

the chiral derivative $l_\mu = \partial_\mu U U^\dagger$, $U \in SU(2)$ or $U \in SU(3)$ - unitary matrix depending on chiral fields, m_π is the pion mass, F_π - the pion decay constant known experimentally, e - the only parameter of the model in its minimal variant proposed by Skyrme [1].

The mass term $\sim F_\pi^2 m_\pi^2$, changes asymptotics of the profile f and the structure of multiskyrmions at large B , in comparison with the massless case. For the $SU(2)$ case

$$U = \cos f + i (\vec{n} \vec{\tau}) \sin f, \quad (6)$$

the unit vector \vec{n} depends on 2 functions, α, β . Three profiles $\{f, \alpha, \beta\}(x, y, z)$ parametrize the 4-component unit vector on the 3-sphere S^3 .

The topological soliton (skyrmion) is configuration of chiral fields, possessing topological charge identified with the baryon number B [1]:

$$B = \frac{1}{2\pi^2} \int s_f^2 s_\alpha I[(f, \alpha, \beta)/(x, y, z)] d^3 r, \quad (7)$$

where I is the Jacobian of the coordinates transformation, $s_f = \sin f$. So, the quantity B shows how many times the unit sphere S^3 is covered when integration over 3-dimensional space R^3 is made.

The chiral and flavor symmetry breaking term in the lagrangian density depends on kaon mass and decay constant m_K and F_K ($F_K/F_\pi \simeq 1.23$ from experimental data):

$$\begin{aligned} L^{FSB} = & \frac{F_K^2 m_K^2 - F_\pi^2 m_\pi^2}{24} \text{Tr}(U + U^\dagger - 2)(1 - \sqrt{3}\lambda_8) - \\ & - \frac{F_K^2 - F_\pi^2}{48} \text{Tr}(U l_\mu l_\mu + l_\mu l_\mu U^\dagger)(1 - \sqrt{3}\lambda_8) \end{aligned} \quad (8).$$

This term defines the mass splittings between strange and nonstrange baryons (multibaryons), modifies some properties of skyrmions and is crucially important in our consideration.

As we have stressed previously, the great advantage of the CSA is that multibaryon states — nuclei, hypernuclei ... — can be considered on equal footing with the $B = 1$ case. Masses, binding energies of classical configurations, the moments of inertia $\Theta_I, \Theta_J, \dots$, the Σ -term (we call it Γ) and some other characteristics of chiral solitons contain implicitly information about interaction between baryons. Minimization of the mass functional M_{class} provides 3 profiles $\{f, \alpha, \beta\}(x, y, z)$ and allows to calculate moments of inertia, etc .

4 Mass formula for multibaryons quantized in $SU(2)$

In the $SU(2)$ case, the rigid rotator model (RRM) used at first in [3] for the $B = 1$ case, is most effective and successfull. It allowed to describe successfully the properties of nucleons, $\Delta(1232)$ -isobar, as well as many properties of light nuclei [11], and also mass splittings of nuclear isotopes, including neutron rich nuclides with atomic numbers up to ~ 30 [9].

When the basic classical configuration possesses definite symmetry properties, the interference between iso- and usual space rotations becomes important. We consider here first an example of the axially symmetrical configuration which is believed to provide the absolute minimum of the classical static energy (mass) for the $B = 2$ case. The mass formula for the axially symmetric configuration has been obtained first for the nonstrange states in [4] and, in greater detail somewhat later, in [5]:

$$M(B, I, J, \kappa) = M_{cl} + \frac{I(I+1)}{2\Theta_I} + \frac{J(J+1)}{2\Theta_J} + \frac{\kappa^2}{2} \left(\frac{1}{\Theta_{I,3}} - \frac{1}{\Theta_I} - \frac{4}{\Theta_J} \right), \quad (9)$$

where $\kappa = I_3^{bf}$, I^{bf} and J^{bf} are body-fixed isospin and spin of the system, and the relation takes place $J_3^{bf} = -2I_3^{bf}$ as a consequence of the generalized axial symmetry of the $B = 2$ classical configuration (see Eq. (10)).

This formula is in agreement with known quantum mechanical formulas for the energy of axially symmetrical rotator [33]. The classical characteristics of the lowest baryon numbers states — moments of inertia $\Theta_I, \Theta_J, \Theta_{I,3}$, which enter formula (9), as well as other quantities, necessary for calculating the spectrum of $SU(3)$ quantized states, are given in Table 1.

B	Θ_I	Θ_J	Θ_3	Θ_S	Γ	ω_S	μ_S
1	5.55	5.55	5.55	2.04	4.80	306	3.165
2	11.47	19.74	7.38	4.18	9.35	293	3.081
3	14.4	49.0	14.4	6.34	14.0	289	3.066
4	16.8	78.0	20.3	8.27	18.0	283	2.972

Table 1. Characteristics of classical skyrmion configurations which enter the mass formulas for multibaryons. The numbers are taken from [34, 35]: moments of inertia Θ and Σ -term Γ - in units GeV^{-1} , ω_S - in MeV , μ_S is dimensionless (see next sections for explanation)⁵. Parameters of the model $F_\pi = 186 MeV$; $e = 4.12$ [16, 34, 35].

The rational map approximation [36] simplifies considerably calculations of various characteristics of classical multiskyrmions presented in Table 1⁶. The value of Θ_J in Table 1 for the baryon numbers $B = 3$ and 4 is the average one of the diagonal elements of the orbital inertia tensor.

Here in Tables 2,3 we present for completeness the result of the calculation of dibaryons spectrum according to the above formula. Many of these results have been obtained previously in [5]; unlike [5] we pretend to calculate the differences of masses of states with different quantum numbers, not the absolute values of masses which are controlled by the loop corrections and/or so called Casimir energy which has been calculated approximately for the $B = 1$ case, see [37, 38]. For our choice of the model parameters the mass differences presented in Tables 2,3 are somewhat smaller (by few tens of MeV) than the mass differences which can be extracted from results of [5] obtained with parameters of the paper [3].

I	J	κ	Content	ΔE	$\Delta E (MeV)$
1	0	0	$NN(^1S_0)$	0	
0	1	0	$NN(^3S_1)$	$1/\Theta_J - 1/\Theta_I$	-36
*1	1	0	$NN\pi?$	$1/\Theta_J$	51
1	2	0	$NN(^1D_2); \Delta N(^5S_2)$	$3/\Theta_J$	153
0	3	0	$NN(^3D_3); \Delta\Delta(^7S_3)$	$6/\Theta_J - 1/\Theta_I$	219
2	1	0	$\Delta N(^3S_1); \Delta N(^3D_1)$	$1/\Theta_J + 2/\Theta_I$	225
3	0	0	$\Delta\Delta(^1S_0); NN\pi\pi$	$5/\Theta_I$	435
2	4	2	$\Delta N(^5D_4); NN\pi$	$2/\Theta_J + 1/\Theta_I + 2/\Theta_{I,3}$	462

Table 2. The quantum numbers, possible hadronic content and the energy (in MeV) of positive parity states above the singlet NN scattering state with $I = 1$, $J = 0$.

⁵In some formulas we add lower index B for all quantities to emphasize dependence on baryon number, e.g. $\mu_{S,B}$.

⁶Explicit expressions for the quantities shown in this table can be found in [35] and [11].

As it was shown first in [5], the parity of such states is

$$P_{ax,B=2} = (-1)^\kappa. \quad (10)$$

The deuteron-like state ($I = 0, J = 1$) has energy by 36 MeV lower than the NN scattering state $I = 1, J = 0$ [4, 5]⁷, so, the value $\sim 40 \text{ MeV}$ can be considered as uncertainty of our predictions of masses in the $SU(2)$ case.

The coefficient after κ^2 in eq. (9) is negative, therefore, states with maximal possible value of $|\kappa|$ at fixed I, J have the lowest energy, linearly dependent on I and J after cancellation of quadratic terms:

$$E_{kin} = \frac{I}{2\Theta_I} + \frac{J}{2\Theta_J} + \frac{I}{2\Theta_{I,3}}. \quad (11)$$

This formula is valid for negative parity states with $I = 1, J = 2, \kappa = \pm 1$, or, generally, $J = 2I, \kappa = \pm I$.

I	J	κ	Content	ΔE	$\Delta E (\text{MeV})$
1	2	± 1	$NN(^3P_2)$	$1/\Theta_J - 1/2\Theta_I + 1/2\Theta_{I,3}$	75
1	3	± 1	$NN(^3P_3, ^3F_3)$	$4/\Theta_J - 1/2\Theta_I + 1/2\Theta_{I,3}$	229
2	2	± 1	$\Delta N(^3P_2); NN\pi$	$1/\Theta_J + 3/2\Theta_I + 1/2\Theta_{I,3}$	249
2	3	± 1	$\Delta N(^5P_3); NN\pi$	$4/\Theta_J + 3/2\Theta_I + 1/2\Theta_{I,3}$	402
2	4	± 1	$\Delta N(^3F_4); NN\pi$	$8/\Theta_J + 3/2\Theta_I + 1/2\Theta_{I,3}$	606

Table 3. The quantum numbers, possible hadronic content and the energy of negative parity states above the NN scattering state with $I = 1, J = 0$.

Some comment is necessary concerning the state with $I = J = 1, \kappa = 0$ which is forbidden by Finkelstein - Rubinstein (FR) constraint and cannot decay into the NN -pair due to the Pauli principle. Such a state, if it exists, is an example of elementary particle with $B = 2$, different from ordinary deuteron or singlet scattering state consisting mainly of two nucleons [39]. Such states have been considered earlier in [40, 41] where their masses were found to be higher, greater than 2120 MeV . Experimental situation with possible observation of such states has been described in [42].

The energy of such state, shown in Table 2, does not include the possible difference of Casimir energies (or loop corrections) between FR allowed and FR forbidden states. If this energy is large, this state, as well as the $I = J = 0$ state, should have energy larger than shown in Table 2.

Generally, for multiskyrmions the internal constituents — nucleons, first of all — are not identifiable immediately. Some guess and analysis of quantum numbers are necessary for this purpose. A possible hadronic content of dibaryon states is shown in Tables 2, 3. Evidently, states with the value of isospin $I \geq 2$ cannot be made of 2 nucleons only, additional pions are needed, or Δ instead of some nucleons. By same reason the states with $I \geq 2$ cannot be observed in nucleon-nucleon interactions. The states with isospin 0 or 1 could appear as some enhancements in corresponding partial wave of the NN scattering amplitude.

For configuration with baryon number $B = 3$ the symmetry properties of the classical configuration important for quantization have been established first by L.Carson [6] and

⁷The measured value of deuteron binding energy is $\epsilon_d \simeq 2.2 \text{ MeV}$. Within the CSA the deuteron-like state is lower in energy than the singlet NN scattering state because the orbital inertia Θ_J is considerably, by a factor 1.5, greater than the isotopic moment of inertia Θ_I , see Table 1. This remarkable property takes place in all known variants of the CSA.

exploited recently in [11]. As a consequence of the symmetry properties of classical $B = 3$ configuration which has characteristic tetrahedral shape, the equality between body fixed spin and isospin takes place, $K = L$. The parity of quantized states equals [6]

$$P = (-1)^{(K_3+L_3)/2} = (-1)^{M_3/2}. \quad (12)$$

The analysis and interpretation of the $B = 3$ states is more complicated than the $B = 2$ states, and only few of them were considered in [6, 11].

5 Spectrum of multibaryons with strangeness in the rigid oscillator model

The observed spectrum of states is obtained by means of quantization procedure and depends on the quantum numbers and characteristics of skyrmions presented in Table 1.

Within the bound state model (BSM) [14, 15, 16] antikaon is bound by the $SU(2)$ skyrmion. The mass formula takes place

$$M = M_{cl} + \omega_S + \omega_{\bar{S}} + |S|\omega_S + \Delta M_{HFS} \quad (13)$$

where flavor and antiflavor excitation energies

$$\omega_S = N_c(\mu_S - 1)/8\Theta_S, \quad \omega_{\bar{S}} = N_c(\mu_S + 1)/8\Theta_S, \quad (14)$$

$$\mu_S = \sqrt{1 + \bar{m}_K^2/M_0^2} \simeq 1 + \frac{\bar{m}_K^2}{2M_0^2},$$

$$M_0^2 = N_c^2/(16\Gamma\Theta_S) \sim N_c^0, \quad \mu_S \sim N_c^0, \quad (15)$$

N_c is the number of colors of underlying QCD.

The hyperfine splitting correction depending on hyperfine splitting constants c_S , \bar{c}_S , isospin, "strange isospin" I_S and angular momentum J equals in the case when interference between usual space and isospace rotations is negligible or not important, is

$$\Delta M_{HFS} = \frac{J(J+1)}{2\Theta_J} + \frac{c_S I_r(I_r+1) - (c_S - 1)I(I+1) + (\bar{c}_S - c_S)I_S(I_S+1)}{2\Theta_I} \quad (16)$$

The hyperfine splitting constants are equal

$$c_S = 1 - \frac{\Theta_I}{2\Theta_S\mu_S}(\mu_S - 1), \quad \bar{c}_S = 1 - \frac{\Theta_I}{\Theta_S\mu_S^2}(\mu_S - 1), \quad (17)$$

Strange isospin equals $I_S = 1/2$ for $S = \pm 1$, for negative strangeness in most cases of interest $I_S = |S|/2$ which minimizes this correction (but generally it can be not so). We recall that body-fixed isospin $\vec{I}^{bf} = \vec{I}_r + \vec{I}_S$, \vec{I}_r is the isospin of skyrmion without added antikaons. It is quite analogous to the so called "right" isospin within the rotator quantization scheme. When $I_S = 0$, i.e. for nonstrange states, $I = I_r$ and this formula goes over into $SU(2)$ formula for multiskyrmions

$$E_{kin} = \frac{J(J+1)}{2\Theta_J} + \frac{I(I+1)}{2\Theta_I}, \quad (18)$$

where we neglect the interference terms [35, 11]. Correction $\Delta M_{HFS} \sim 1/N_c$ is small at large N_c , and also for heavy flavors [14, 35].

For the case of classical state with generalized axial symmetry an additional term appears

$$\Delta E^{axial} = \frac{\kappa^2}{2} \left[\frac{1}{\Theta_3} - \frac{1}{\Theta_I} - \frac{4}{\Theta_3} \right] = \kappa^2 \delta(\Theta), \quad (19)$$

which differs for states with different parities (different κ , see Tables 1,2).

The mass splitting within $SU(3)$ multiplets is important and convenient for us here since the unknown for the $B \geq 1$ solitons Casimir energy cancels in the mass splittings. For the difference of energies of states with strangeness S and with $S = 0$ which belong to the same multiplet (p, q) we obtain using the above expressions for the constants c_S and \bar{c}_S

$$\begin{aligned} \Delta E(p, q; I, J, S; I_r, J_0, 0) = & |S| \omega_S + \frac{\mu_{S,B} - 1}{4\mu_{S,B} \Theta_{S,B}} [I(I+1) - I_r(I_r+1)] + \\ & + \frac{(\mu_{S,B} - 1)(\mu_{S,B} - 2)}{4\mu_{S,B}^2 \Theta_{S,B}} I_S(I_S+1) + \frac{1}{2\Theta_J} [J(J+1) - J_0(J_0+1)] + (\kappa^2 - \kappa_0^2) \delta(\Theta), \end{aligned} \quad (20)$$

if the underlying classical configuration possesses axial symmetry. For arbitrary strangeness $I_S \leq |S|/2$, and $J_0 = J$ if these states belong to the same $SU(3)$ multiplet. The values of the quantities which enter above formulas are shown in Table 1.

6 Dibaryons with strangeness

Strange dibaryons have attracted much attention beginning with pioneer papers [43, 40, 41, 44, 45]. Recent discussion of this topic and many important references can be found in [28]. Here we do not discuss the $S = -2$ H-dibaryon [43] which is the $SU(3)$ singlet and appears as the $SO(3)$ soliton within the chiral soliton approach [46, 47].

For completeness we present here the former results by B.Schwesinger et al [48] for energies of different strange dibaryons within the soft rotator model with $SU(3)$ configuration mixing.

<i>multiplet</i>	$\{\bar{10}\}$	$\{27\}$	$\{\bar{10}\}$	$\{27\}$	$\{27\}$	$\{27\}$	$\{27\}$	$\{35\}$	$\{28\}$
S, I	-1, 1/2	-1, 1/2	-2, 1	-2, 0	-3, 1/2	-3, 3/2	-4, 0	-5, 1/2	-6, 0
<i>state</i>	ΛN	ΛN	ΞN	$\Lambda\Lambda$	$\Lambda\Xi$	$\Sigma\Xi$	$\Xi\Xi$	$\Xi\Omega$	$\Omega\Omega$
ΔE^{SRM}	30	70	100	110	140	90	150	40	30

Table 4. The energy above threshold ΔE in MeV for dibaryons with $J^P = 0^+$, different values of strangeness S and isospin I . The $SU(3)$ multiplet, which the main component of the dibaryon configuration belongs to, is indicated in the upper line. Calculations made according to the soft rotator model [48].

As can be seen from Table 4, we did not predict in [48] the bound states of dibaryons, all states of lowest energy shown in this Table are above the corresponding two-baryon thresholds (for consistency we took theoretical values of baryon masses which do not coincide with empirical values). These lowest states can be and should be interpreted as virtual states, or scattering states similar to the $(NN) {}^1S_0$ scattering state, the so called singlet deuteron. Presence of such states leads to the enhancement of scattering cross section in corresponding channel, as seen in the ΛN or $\Lambda\Lambda$ data, (see, e.g. [28]). In view of considerable numerical uncertainty of these results there remains still a chance that nearest to threshold dibaryons can be bound.

In previous publication on this subject [49] we obtained bound dibaryons, but the poorly known Casimir energies of the order of N_c^0 [37, 38] (discussed already in this paper in connection with nonstrange dibaryons) have not been taken into account in [49]. In fact, we should write for baryons

$$M_1(p, q; Y, I) = M_1^{class} + \Delta M_1(p, q; Y, I) + M_1^{Cas} \quad (21)$$

and for dibaryons (multibaryons in general case)

$$M_2(p, q; Y, I) = M_2^{class} + \Delta M_2(p, q; Y, I) + M_2^{Cas}. \quad (22)$$

$\Delta M(p, q; Y, I)$ is the quantum numbers dependent quantum correction, hypercharge $Y = B + S$, $M^{Cas} \sim N_c^0$ is the Casimir energy or loop correction. When we calculated the energy (mass) difference

$$\begin{aligned} \Delta M &= M_1(p_1, q_1; Y_1, I_1) + M_1(p_2, q_2; Y_2, I_2) - M_2(p, q; Y, I) = \\ &= 2M_1^{class} - M_2^{class} + \Delta M_1(p_1, q_1; Y_1, I_1) + \Delta M_1(p_2, q_2; Y_2, I_2) - \Delta M(p, q; Y, I) + \\ &\quad + 2M_1^{Cas} - M_2^{Cas}, \end{aligned} \quad (23)$$

in [49] we ignored the term $2M_1^{Cas} - M_2^{Cas}$ and obtained strong binding due to large contribution of $\Delta M_1(p_1, q_1; Y_1, I_1)$ and $\Delta M_1(p_2, q_2; Y_2, I_2)$. This very large binding seemed apparently unrealistic, and reasonable way out of this situation appeared when it was recognized that the contributions of the order of N_c^0 due to poorly known loop corrections, or Casimir energy make large negative contribution both to M_1 [37, 38] and, probably, to M_2 . To obtain the NN singlet scattering state on the right place, we should have [48]

$$2M_1^{Cas} - M_2^{Cas} \simeq -820 \text{ MeV}, \quad (24)$$

for the choice of parameters made in [48], and results shown in Table 4 follow immediately. Up to now these contributions to classical masses of skyrmions were calculated very approximately only for the unit ($B = 1$) skyrmion [37, 38]. These contributions are negative $M_1^{Cas} \sim -1 \text{ GeV}$, i.e. they act in right direction. For larger baryon numbers Casimir energy has not been calculated yet, because it is very nontrivial computational problem.

Prediction of the $S = -3$ dibaryons with $(J^P; I) = (1^+, 2^+; 1/2)$ below the $\Lambda\Xi$ threshold has been made long ago by Goldman et al [44] within a variant of the MIT bag model. Recently strong attraction was found in some two-baryon channels with strangeness $S = -3$ and -4 , in the leading order of chiral effective field theory, suggesting the possible existence of bound states [26]. Latest studies of strange dibaryons within quark models are presented in [27] and references therein.

To get spectrum of strange dibaryons in our chiral soliton approach we should transform basic formula (16) for the quantum correction to the energy of multiskyrmions to

$$\begin{aligned} \Delta M &= |S|\omega_S + \frac{1}{2\Theta_I} \left[cI_r(I_r + 1) + (1 - c)I(I + 1) + (\bar{c} - c)I_S(I_S + 1) \right] + \\ &\quad + \frac{J(J + 1)}{2\Theta_J} + \frac{\kappa^2}{2} \left(\frac{1}{\Theta_{I,3}} - \frac{1}{\Theta_I} - \frac{4}{\Theta_J} \right), \end{aligned} \quad (25)$$

and $B = 2$ in all quantities Θ , ω_S to be taken from Table 1. I_r (the right isospin within the rigid rotator quantization scheme) is the isospin of the nonstrange state, $I_S \leq |S|/2$ is the isospin carried by strange mesons, and the observed isospin $\vec{I} = \vec{I}_r + \vec{I}_S$. For $S = 0$ and $I = I_r$ we recover the above formula (9) for the quantum correction to the $SU(2)$ quantized dibaryons.

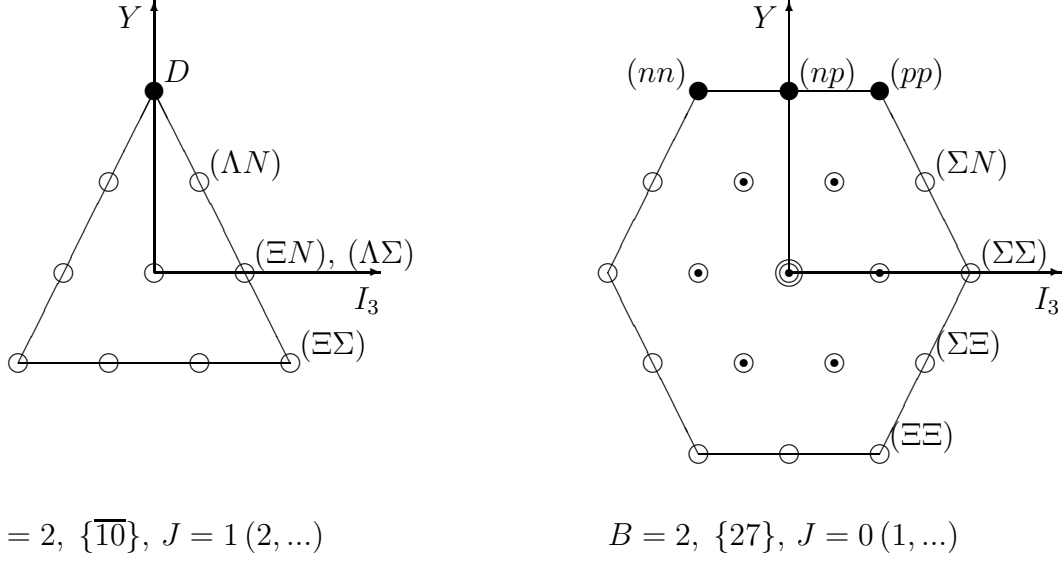


Figure 1: $I_3 - Y$ diagrams of multiplets of dibaryons $B = 2$. Virtual levels (scattering states) are shown in brackets, e.g. (ΛN) scattering state which appears as near threshold enhancement.

For the difference of energies of states which belong to antidecuplet and singlet $(NN)^1S_0$ state we obtain

$$\begin{aligned}
 E(0, 3; I, J, S) - E(2N, ^1S_0) = & |S|\omega_S + \frac{\mu_{S,B} - 1}{4\mu_{S,B}\Theta_{S,B}} I(I+1) + \\
 & + \frac{(\mu_{S,B} - 1)(\mu_{S,B} - 2)}{4\mu_{S,B}^2\Theta_{S,B}} I_S(I_S + 1) + \frac{1}{2\Theta_J} J(J+1) - \frac{1}{\Theta_I} + \kappa^2\delta(\Theta),
 \end{aligned} \tag{26}$$

and in our case of $S = -1$ we should take $I_S = 1/2$. The only allowed possibility for κ is $\kappa = 0$, because $I_r = 0$. Numerical values of dibaryons energies are given for several lowest states in Table 5.

I_r	J	I	S	κ	$\Delta E(MeV)$
0	1	1/2	-1	0	289
0	2	1/2	-1	0	392
0	3	1/2	-1	0	546

Table 5. $B = 2$ states: set of quantum numbers and the energy above the NN scattering state for the $S = -1$ states with $I_r = 0$ and different values of spin, to be ascribed to antidecuplet, $(p, q) = (0, 3)$, shown in Fig.1a.

The state with $J = I_r = 0$, $S = -1$, $I = 1/2$, not shown in Table 5, has energy $\Delta E(0, 0, 1/2, -1) \simeq 238 MeV$. but this state cannot belong to the antidecuplet containing deuteron with $J = 1$.

For dibaryon states which belong to $\{27\}$ -plet we can use Eq. (20) with $I_r = 1$, $I_S = 1/2$, $J_0 = 0$, $\kappa_0 = 0$. Numerical results are shown in Table 6.

We would like to stress again that we are not fitting — here and previously — the absolute values of masses of nucleons, hyperons and nuclei (in difference from papers [3, 11]) because they are controlled by poorly known loop corrections or Casimir energy (see discussion of Eq. (24)).

with $\Theta'_J = (\Theta_J \Theta_I - \Theta_{int}^2)/(\Theta_I - \Theta_{int})$, all quantities should be taken from Table 1 for $B = 3$, $\Theta_{int} \simeq -9.4 \text{ Gev}^{-1}$.

For the ground $B = 3$ state the $SU(3)$ multiplet with $(p, q) = (1, 4)$, $I_r = J_0 = 1/2$ ($\overline{\{35\}}$ -plet) is shown in Fig.3a. Fig.3b for even B -numbers is included for illustration. The equality $J_0 = I_r$ follows from the symmetry properties of the $B = 3$ classical configuration which has tetrahedral form. see [6].

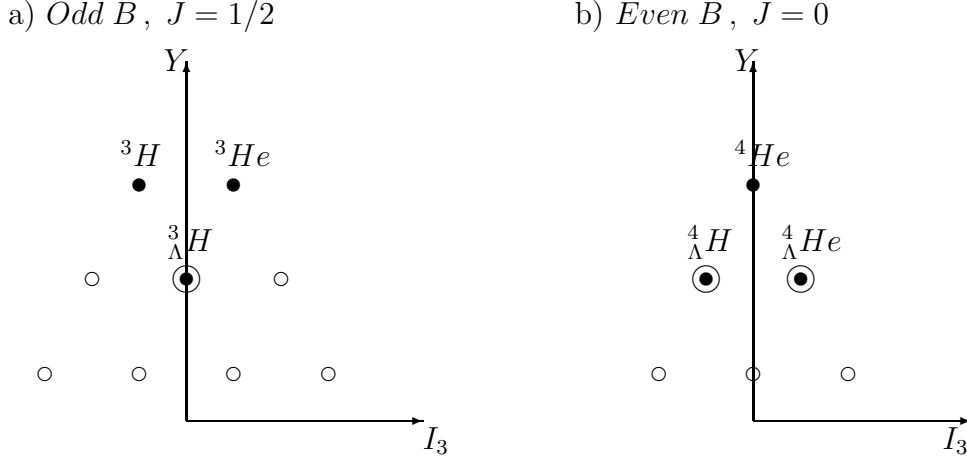


Figure 3: (a) The location of the isoscalar ground state (shown by double circle) with odd B -number and $S = -1$ in the upper part of the $(I_3 - Y)$ diagram. (b) The same for isodoublet states with even B . The case of light hypernuclei ${}_{\Lambda}H$ and ${}_{\Lambda}He$ is presented as an example. The lower parts of diagrams with $Y \leq B - 3$ are not shown here.

I_r	J	I	M	$\Delta E(\text{MeV})$
1/2	1/2	0	0	279
1/2	1/2	1	0	321
3/2	3/2	1	0	378
3/2	3/2	2	0	462
3/2	3/2	1	3	302
3/2	3/2	1	2	348
3/2	3/2	2	2	432
5/2	5/2	2	4	421
5/2	5/2	2	3	482

Table 7. Some of possible $B = 3$ -states: set of quantum numbers and the energy above the NNN threshold for states with $I_r = 1/2, 3/2$ and $5/2$, strangeness $S = -1$ and different values of spin, isospin, and parity, $M = M_3$.

Our results for $S = -1$ excited tribaryons are presented in Table 7. The lowest in energy state with $J = I_r = 1/2$, $I = M = 0$ can be naturally interpreted as ${}^3_{\Lambda}H$ hypernucleus. States with $J = 3/2$ and $5/2$ should belong to other $SU(3)$ multiplets.

These results should be considered as preliminary; further studies of this issue are desirable, also for greater baryon numbers, $B \geq 4$.

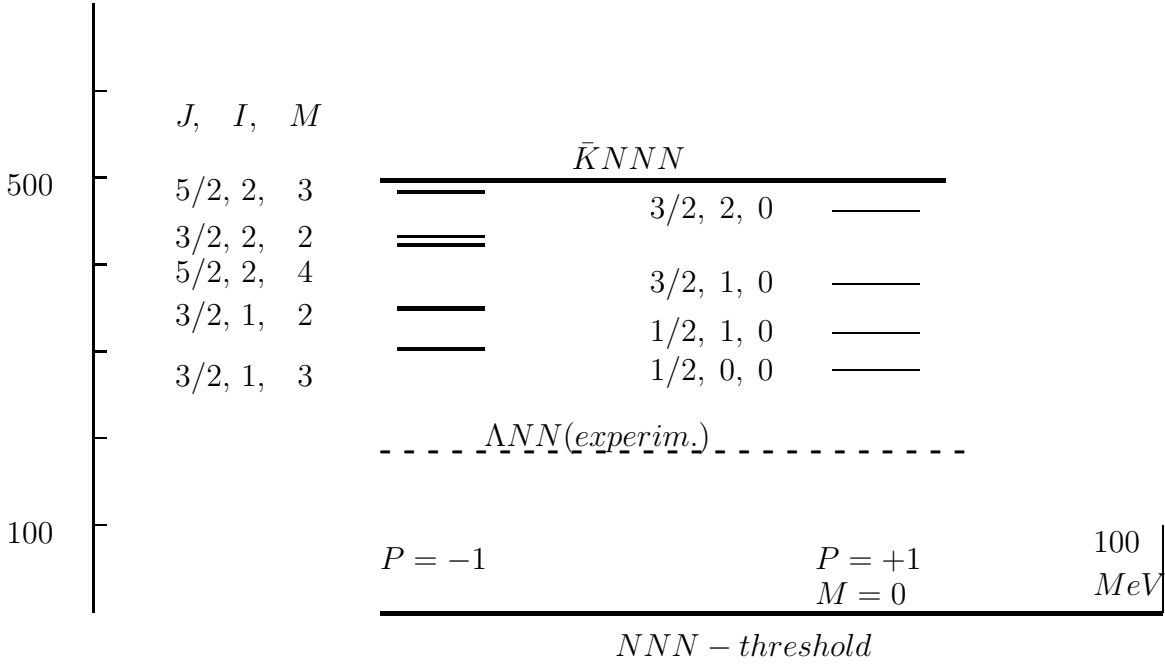


Figure 4: Some of the $B = 3$ rotationally excited states (above the NNN threshold) with negative strangeness $S = -1$, different isospin and spin, negative and positive parities, $I_r = J$.

The restriction on allowed isospin of non-exotic states (i.e. the states without additional quark-antiquark pairs) takes place: $I \leq (3B + S)/2$, and for antikaon-nuclei bound states, evidently, $I \leq (B + 1)/2$. Second restriction becomes stronger for $B \geq 2$, so, only states with not too large isospin can be interpreted as antikaon-nuclei bound states. Generally, rotational excitations have additional energy $\Delta E = J(J + 1)/2\Theta_J$. The orbital inertia grows fast with increasing baryon (atomic) number, $\Theta_J \sim B^p$, p is between 1 and 2. By this reason the number of rotational states becomes large for large baryon numbers.

8 Summary and conclusions

To summarize, we have considered here rotational-type excitations of the $S = -1$ baryonic systems (nuclei) with baryon number $B = 2$ and, partly, $B = 3$ using the chiral soliton approach. It was assumed that during the collective motion the shape of the basic classical configuration is not changed. We did not consider the vibration-breathing excitations which are possible as well. For the baryon number 1 it was possible to describe in this way some properties of the negative parity $\Lambda(1405)$ -state [30]. For the case of dibaryons some nonstrange states have been considered in [32], although numerical results have not been presented. Similar states should exist for strange multibaryons, but numerical computations are extremely complicated.

We investigated only one of several possible variants of multiskyrmions quantization in the $SU(3)$ extension of the chiral soliton model, the rigid rotator/oscillator variant. The rich spectrum of strange multibaryons is predicted within this approach, with positive as well as with negative parities. There is rigorous theoretical statement that at small value of kaon mass there should be quantized states with strangeness -1 with energy below the $NN... + \bar{K}$ threshold.

The existence of strange excited nuclear states which could be interpreted as bound antikaon-nuclear states within the CSA seems to be quite natural and not unexpected. However, when the energy below the threshold becomes large, interpretation of such states as the bound state of antikaon and corresponding nonstrange nucleus becomes less straightforward, due to

increase of the weight of other components, first of all, containing hyperons. For realistic value of the kaon mass some states are predicted, but with considerable numerical uncertainty. In view of these uncertainties, experimental investigations could play decisive role. Since several such states are expected in the energy gap equal to one kaon mass, good enough experimental resolution in their energy (mass) of the observed states is of great importance. Another option can be that there are several wide overlapping states, and in this case better resolution will not help much.

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